ON DETERMINATION OF YARDSTICKS OF ADDITIONAL PRODUCTION

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INTRODUCTION

Target setting is an essential prerequisite for developmental planning. These targets are fixed for various commodities according to the physical requirements and technical and economic feasibility within the overall plan allocation.

In India the method adopted for fixation of targets in the agricultural sector is Production Potential or Yardstick approach. For an evaluation of the contribution of each developmental measure to the additional production, a knowledge of Input-Output Coefficient in physical units or yardstick, based on technical data suitably collected from experiments, surveys etc. is necessary.

Panse, Abraham and Leelavathi (1964) have evaluated the yardsticks of additional production from the use of nitrogenous and phosphatic fertilizers, farm yard manure and green manure. Available data on the response to irrigation, soil conservation, combinations of some of the inputs have been presented. The yardsticks of additional production from the use of potassic fertilizers have been worked out by Seth, Jha and Leelavathi (1968).

In the present paper the method of estimation of yardsticks of additional production from large scale experimental data from trials on cultivators' fields is investigated. The nature and magnitude of errors in the estimated yardsticks arising from sampling and bias due to aggregation and specification are discussed.

^{1.} On deputation to Iran on F.A.O. assignment.

2. MATERIAL AND METHODS

The estimates of yardsticks of additional production from the use of nitrogenous and phosphatic fertilizers on indigenous varieties of rice and wheat are presented. These are based on the data from experiments laid out on randomly selected cultivators' fields where one of the treatments was the same as normal practices followed by the cultivator and the other treatments super-imposed over the normal practice. These were conducted under the auspices of (i) Stewart's scheme (ii) Bihar manurial trials (iii) Technical cooporation mission (1959, 1959) and (iv) and Coordinated Agronomic Experiments Scheme.

A brief description of the design, spread etc. are given:

(i) Stewarts Scheme:

This was in operation during 1952-55 where three to six plot experiments were conducted in 11 districts of seven states: Bihar, Bhopal, Bombay, Madras, Punjab, Pepsu and West Bengal mainly on rice and wheat.

(ii) Bihar manurial trials:

An extensive series of fertilizer trials covering all districts of the state was taken up in Bihar on important crops viz. rice, wheat, maize, marua and gram in four to seven plot experiments. The results from 1949-56 are included in this paper.

(iii) Technical Cooperation mission:

A series of five to six plot experiments were conducted during 1953-55 under this scheme in 22 community projects located in different regions of the country covering the rice and wheat areas.

(iv) All India Coordinated Agronomic Experiments Scheme:

This was taken up in all states as a continuation of technical cooperation mission scheme. The experiment had five to eight plots covering important cereals, pulses, oilseed and cash crops. Results from 1958-61 are considered.

The extent of data considered was as follows (see Table 1).

The estimation of the standard error of the yardsticks of additional production from fertilizer use is illustrated with the data obtained from the coordinated agronomic experiments scheme. The effect of the distribution pattern of fertilizer use in cultivators' holding

Table 1

Description of data

Input	Crop	No. of experiments	Nature of treatments
Nitrogen	Rice	10773	10-70 kg of nutrient/ha
(N)	Wheat	7373	, ,
Phosphorus	Rice	6344	, ,,
(P ₂ O ₅)	Wheat	4840 ,	·

on the estimate of yardstick is shown based on the data from surveys on farmers' fertilizer practices (1960, 1964).

Abraham and Rao (1966) have studied the dose-response relationship by fitting suitable mathematical functions and have indicated the suitability of quadratic function. Abraham (1966) has pointed out that with the low level of application at present, the increased quantity of fertilizer made available in the country will, to a large extent go for application of new areas rather than increasing the level of application of fertilizer and therefore the error resulting from the use of average response instead of the marginal response is small.

2.2 Estimation of yardsticks of additional production

The procedure of estimation of yardsticks of additional production basically consisted of choosing a standard dose for a given crop: reducing the responses to different doses to that of standard dose with the help of quadratic response curves fitted to the data; estimating the average standardised response for a region or state and converting this response as output per unit input.

Where response curves were available from more than one series at the district level, a weighted average response curve was obtained with the number of experiments on which each series is based, as weights.

The conversion factor (also known as standardisation factor) were worked out for various levels at $\frac{\mathbf{Y}_s}{\mathbf{Y}_x}$ where \mathbf{Y}_s and \mathbf{Y}_x are the responses to standard dose and dose x respectively.

The responses to diverse levels multiplied by the corresponding standardisation factor gives the standardised response. The method is illustrated with data from Hazaribagh district (Bihar State) on rice, in tables 2 and 3.

TABLE 2
Standardisation of respnses

Soi	urce of data	periments n	Dose of aitrogen in kg/ha	Response in q ha.	Standard response in q ha.	
1.	Manurial trials in Bihar	188	33.6	3.33	2.54	2.54
2.	T.C.M. Scheme	146*	22.4 44.8	2.00 3.47	2.00 2.31	2.16
3.	Coordinated agronomic	63 (A type)	22.4	3.73	3.73	3.73
	experiments scheme	59°° (B type)	22.4 44.8	2.93 3.27	2.93 2.18	2 55

Note:—Response curves.: - $^{\circ}Y=0.101116 \times -0.000528 \times ^{2}$

** $Y=0.188616 x-0.002580 x^2$

Overall curve $Y=0.126299 \text{ x}-0.001119 \text{ x}^2$

TABLE 3

Evalution of standardisation factor

Dose of nitro- gen in kg ha	Value of overall curve in q/ha (cf. Table 2)	Standardisation' factor $Ys Y_x$
11,2	1.27	1.7874
22.4	2.27	1.0000
33.6	2.98	0.7617
44.8	3.41	0.6657

A dose of 22.4 kg N/ha is taken as the standard dose.

The average standardised responses from all the series are considered and the district response is obtained as a weighted average of such responses, the weights being the number of experiments on which each series is based. An arithmetic mean of the district responses over the years is taken as the average district response. The standardised mean response for the state, is, in turn, an arithmetic mean of the average responses over the districts.

Let

 $\overline{\overline{Y}}$ = average standardised response in kg/ha for the state.

s = standard dose in kg/ha

The yardstick of additional production is worked out as

$$R' = \frac{\overline{\overline{Y}}}{s}$$

In working out the yardsticks of additional production (Panse et al, 1964) the standard dose taken for each input is the one which is generally recommended to the farmers for the crop. A level of 22.4 kg nitrogen (N) and phosphorus (P₂O₅) per hectare was taken as the standard dose for rice and wheat. The estimates are given in table 4.

TABLE 4
Estimates of yardsticks of additional production, their standard errors and confidence intervals

State	No. of experi- ments	Yardsticks of addition production	al %	Standard error•	Confi inter	idence val*
		7/		-		
		Use of	nitrogen o	n rice		
Andhra Pradesh	1232	10.75	7.47	1.70	7.41	to 14.11
Assam	597	10.71	10.83	1.91	6.96 t	o 14.46
Bihar	3437	11.61	2.69	0.60	10.40 t	o 12.81
Kerala	783	12.10	11.44	3.06	6.12 t	0 18.08
Madras	741	10.40	. 5.15	0.53	, 9.37 ·t	0 11.43
. Madhya Prade	sh 412	13.08	7.85	1.57	10.00 t	0 16.16
Mysore	887	11.29	4.74	0.86	,9.60t	

TABLE 4 (Contd.)

State	No. of experi- ments	Yardsitcks of addition production	ial %	Stand error		Confidence interval*
Punjab	216	12.14	5.51	1.24	9.69	to 14.60
Uttar Pradesh	1235	10.22	2.18	0.40	9.42	to 11.03
West Bengal	1233	. 8.57	6.25	1.02	6.56	to · 10.58
		Use of phos	phorus on r	ice		
Andhra Pradesh	667	6.70	8.00	1.02	4.6	8 to 8.70
Assam	312	10.40	6.01	0.89	8.6	6 to 12.14
Bihar	2 797	8.44	4.76	0.82	6.83	to 10,04
Kerala	403	9.37	1.90	0.20	8.97	to 9.78
Madras	286	7.45	14.97	1.97	3.57	to 11.34
Madhya Pradesh	219	8,39	19.15	2.61	3,30	to 13.48
Mysore	444	8.26	8.65	1.40	5.49	to 11.03
Punjab	106	6.07	12.50	1.48	3.17	7 to 8.97
Uttar Pradesh	614	5.80	6.15	0.66	4.51	to 7.10
West Bengal	496	3.84	13.95	1.01	1.87	to 5.80
	Use d	of nitrogen or	n irrigated	wheat		
Bihar	1995	12.77	6.29	1.70	9.46	to 16.07
Punjab	1361	16.16	8.56	2.97	10.36	to 21.96
Rajasthan	657	9.78	10.04	1.91	6.03	to 13.53
Uttar Pradesh	3360	15.89	2.81	0.95	14.02	to 17.77
	Use o	f phosphorus	on irrigate	d wheat		
Bihar	1783	9.37	2.86	0.28	8.84	to 9.91
Punjab	984	8.26	5.41	0.44	7.41	to 9.11
Rajasthan	381	7.50	10.51	1.45	4.64	to 10.36
Uttar Pradesh	1692	9 55	3.74	0.71	8.17	to 10.94

^{*} In tonnes per tonne of nutrient,

(a) Model

The yardsticks of additional production estimated in the above manner are affected by sampling error, specification error and aggregation errors (Abraham and Leelavathi, 1968). The model may be written as

$$x_{ij} = \mu ... + \alpha_i + \beta_{(i)} + \gamma_{i(i)} + \delta_{ij}$$

(b) Sampling error:

Let n =Number of years (random variable) for which data have been taken.

m = Number of districts (fixed variable) in the state.

 μ_{ij} = True response in the j^{th} year for the i^{th} district

$$(i=1,\ldots,m)$$
$$(j=1,\ldots,n)$$

 x_{ij} = Estimated response in the j^{th} year for the i^{th} district.

It can be seen that

$$\mu_{i.} = \mathop{E}_{j} \left(\mu_{ij} \right)$$

$$\mu_{i,j} = \frac{1}{m} \sum_{i=1}^{m} \mu_{i,i}$$

$$\mu .. = \frac{1}{m} \sum_{i=1}^{m} \mu_i.$$

The true response μ_{ij} can be expressed as

$$\mu_{ij} = \mu... + \alpha_i + \beta_{(j)} + \gamma_{i(j)}$$

Where

$$\alpha_i = \mu_i - \mu_i.$$

$$\beta_{(i)} = \mu_{ij} - \mu_i.$$

$$\gamma_{i(j)} = \mu_{ij} - \mu_i - \mu_{ij} + \mu_i.$$

Further,

$$\sum_{i=1}^{m} \alpha_{i} = 0$$

$$\sum_{j=1}^{m} \beta_{(j)} = 0$$

$$\sum_{i=1}^{m} \gamma_{i(j)} = 0 \text{ for all } j$$

$$\sum_{i=1}^{m} \gamma_{i(j)} = 0 \text{ for all } i$$

The estimated response is thus given by $x_{ij} = \mu ... + \alpha_i + \beta_{(i)} + \gamma_{i(i)} + \delta_{ij}$

Let \overline{X}_{mn} = the estimated average response for the state over m districts and n years.

Then

$$\sum_{m_n=\mu_{\bullet}+\frac{j-1}{m}+\bar{\delta}}^{n}$$

Let

$$Var[\beta_{(i)}] = \sigma^2 \beta$$
$$Var[\delta_{ij}] = \sigma^2 \delta$$

Hence

$$V\left(\bar{x}_{m,n}\right) = \frac{\sigma^2 \beta}{n} + \frac{\sigma^2 \delta}{mn}$$

The analysis of variance can be put in the following form.

Analysis of district responses

Source of variation	D. F. M	ean square	Expectation of mean square
District	(m-1)	M_{α}	$\sigma^2_{\delta} + \sigma^2_{\alpha\beta} + n\sigma^2_{\alpha}$
Year	(n1)	M_{β}	$\sigma^2_{\delta} + m\sigma^2_{\beta}$
Years x districts	(n-1) $(m-1)$	$M_{\alpha\beta}$	$\sigma^2_{\delta} + \sigma^2_{\alpha\beta}$
Error	(n_{δ})	M_{δ}	σ ² δ

The estimates of the components can be obtained as

$$\hat{\sigma}^{2}_{\alpha\beta} = M_{\alpha\beta} - M$$

$$\hat{\sigma}^{2}_{\alpha} = \frac{M_{\alpha} - M_{\alpha\beta}}{n}$$

$$\hat{\sigma}^{2}_{\beta} = \frac{M_{\beta} - M_{\delta}}{m}$$

An estimate of σ^2_{δ} can be obtained as a simple average of the variance in individual years in each district.

The various components are given in table 5.

(c) Confidence interval

Let

 μ . = true response of the r^{th} year.

It can be shown that

$$E(\bar{x}_{mn} - \mu_{r}) = 0$$

$$V(\bar{x}_{mn} - \mu_{r}) = \left(1 + \frac{1}{n}\right)\sigma^{2}_{\beta} + \frac{\sigma^{2}\delta}{mn}$$

It may be assumed that

$$\sqrt{\frac{x_{mn}-\mu_{r}}{\left(1+\frac{1}{n}\right)^{\Lambda_{\sigma^{2}_{\beta}}}+\frac{\sigma^{2}_{\delta}}{mn}}}$$

is distributed as a standard normal deviate. Hence the 95 percent confidence interval can be expressed as

$$\bar{x}_{mn} - 1.96 \sqrt{\left(1 + \frac{1}{n}\right)^{\hat{\Lambda}} \sigma^{2} \beta + \frac{\sigma^{2} \delta}{mn}} \leqslant \mu._{r} \leqslant \bar{x}_{mn} + 1.96$$

$$\sqrt{\left(1 + \frac{1}{n}\right)^{\hat{\Lambda}} \sigma^{2} \beta + \frac{\sigma^{2} \delta}{mn}}$$

(d) Aggregation and specification errors or bias

Let N = no. of fields growing a crop in a district. $a_i = \text{area of the } i^{ih} \text{ field } (i=1,...,N)$

TABLE 5
Estimates of components of variance

State	No. of years	No. of districts	Λ σ ^{2*} β	Λ σ _{2δ} *
	Respons	se to nitrogen on ric	ce	
Andhra Pradesh	4	9	2.2335	3.4275
Assam	2	7	2,3230	2.2496
Bihar	4	11	0.2681	1.4330
Kerala	4	3	7.4163	0.9264
Madras	3	7	0.0000	5.9668
Madhya Pradesh	2	6	1.3797	4.7410
Mysore	4	6	0.4548	4.2087
Punjab	4	5	1.1049	3.3026
Uttar Pradesh	4	14	0.1155	0.8618
West Bengal	3	10	0.7428	1.3843
,, 555 2545	Respons	se to phosphorus on	rice	
Andhra Pradesh	4	9	0.7663	2.7863
Assam	3	6	0.3885	4.8429
Bihar	4	11	0.5025	1.6455
Kerala	4	3	0.0000	0.4747
Madras	3	7	2.6815	6.6479
Madhya Pradesh	2	6	4.1939	6.0120
Mysore	4	8	1.4684	4.5488
Punjab	4	5	1.6489	2.7376
Ulttar Pradesh	3	14	0.3123	0.8086
West Bengal	3	10	0.7294	1.2523
West Dengar		nitrogen on irrigate	d wheat	•
n:1	4	10	2.2281	3.0293
Bihar	4	10	6.9234	5.5026
Punjab	4	4	2.7134	4.2928
Rajasthan Uttar Pradesh	4	17	0.7015	1.2276
Ottat Fradesh	•	phosphorus on irrig	ated wheat	
7 .1	4	9	0.0000	2.7941
Bihar	4	10	0.0000	7.6012
Punjab	4	4	1.4259	5.0581
Rajasthan	4	17	0.3937	1.4768
Uttar Pradesh	4	**		

^{*(}kg of response per kg of nutrient)2

 $x_{io} = \text{no. of units of fertilizer applied in the base year.}$

 $x_{it} = \text{no. of units of fertilizer applied in } t^{th} \text{ year.}$

$$\sum_{i=1}^{N} a_i = A$$

$$(a_i \ x_{it} - a_i \ x_{io}) = \triangle qi$$

$$\sum_{i=1}^{N} \triangle qi = \triangle Q$$

Let $Y_i = b_i x - c_i x^2$ denote the response curve of the i^{th} field y'=b'x-c x^2 the average response curve of the district. and

The aggregate increase in production $\triangle P$ from the use of $\triangle Q$ quantity of fertilizer is given by

$$\triangle P = \sum_{i=1}^{N} a_i b_i (x_{it} - x_{io}) - \sum_{i=1}^{N} a_i c_i (x^2_{it} - x^2_{io})$$

whence the yardstick

$$= \frac{\sum_{i=1}^{N} a_{i}b_{i} (x_{ii} - x_{io}) - \sum_{i=1}^{N} a_{i}c_{i} (x^{2}_{ii} - x^{2}_{io})}{\triangle O} = R \text{ (say)}$$

But the yardstick is calculated as

$$\frac{b's - c's^2}{s} = b' - c's = R' \text{ (say)}$$

Hence

Bias =
$$(R-R')$$

$$= \left[\sum_{i=1}^{N} b_i \left\{ \frac{\triangle q_i}{\triangle Q} - \frac{w_i}{w} \right\} - \sum_{i=1}^{N} c_i \left\{ (x_{it} + x_{io}) \frac{\triangle q_i}{\triangle Q} - s \frac{w_i}{w} \right\} \right] - (i)$$

Where

$$b' = \frac{\sum_{i=1}^{N} w_i b_i}{w}$$

$$\sum_{i=1}^{N} w_i c_i$$

$$c' = \frac{i-1}{w}$$

and

$$w = \sum_{i=1}^{N} wi$$

Systems of weights:

(i) $w_i = \triangle q_i$ and t^{th} should be an printed in page 29 with this system the yardstick would be free from bias if

$$s = x_{it} + x_{io}$$

$$(ii) w_i = a_i \text{ or } 1$$

On the assumption that the rate of application as well as the responses are independent of the size of the field

$$\begin{aligned} \text{Bias} &= \frac{A(N-1)}{N \triangle Q} \{ cov \ (b_i, \ x_{it}) - cov \ (b_i, \ x_{io}) - cov \ (c_i, x_{it}^2) + cov \ (c_i, x_{io}^2) \} \\ &+ \bar{b} \ \left\{ \frac{A}{\triangle Q} (\mu_t - \mu_o) - 1 \right\} \\ &+ \bar{c} \ \left\{ \frac{A}{N \triangle Q} \left[(N-1) \ (\sigma_t^2 - \sigma_o^2) \ + N(\mu_t^2 - \mu_o^2) \right] - s \right\} - (ii) \end{aligned}$$

Where

$$\sigma_o^2 = Var(x_{io}); \quad \sigma_t^2 = Var(x_{it})$$

$$\mu_o = \sum_{i=1}^N x_{io} / N; \quad \mu_t = \sum_{i=1}^N x_{it} / N$$

$$\overline{b} = \sum_{i=1}^N b_i / N; \quad \overline{c} = \sum_{i=1}^N c_i / N$$

The bias can vanish if

- (i) the number of fields are large.
- (ii) covariances of the linear regression coefficient with the rates of application in the base year and the tth year are equal.
- (iii) Covariances of the quadratic regression coefficient with the squares of the rates of application in the base year and the tth year are equal and the standard dose is chosen such that

$$s = \frac{A}{\Lambda O} [\sigma_t^2 - \sigma_o^2 + \mu_t^2 - \mu_o^2]$$

The nature and extent of specification and aggregation errors have been worked out (Abraham and Leelavathi, 1968) utilizing the formula (ii) on the assumption that the number of fields are large; the rate of application of fertilizer is not correlated either with the size of the field or with the magnitudes of linear and quadratic regression coefficients, in which case the bias amounts to

$$-\tilde{c}\left[\frac{A}{\Delta Q}\left\{\sigma_{t}^{2}-\sigma_{o}^{2}+\mu_{t}^{2}-\mu_{o}^{2}\right\}-s\right]$$

TABLE 6 Estimates of bias in yardstick

State	District	Crop	Year	Mean rate of application of nitrogen in kg ha.	Standard deviation of rate of application of nitrogen in kg/ha	Bias
Madras	Coimbatore	Rice	1954-5 1961-6	- ,	23.9 29.9	+7.8
Uttar Pradesh	Barabanki	Rice	1956-5 196 2 -6	-	8.9 10.2	+0.5
_		Wheat	1956-5 1962-6		12.1 14.3	+4.6

- N.B. 1. Standardisation level is taken as 22.4 kg/ha of nitrogen.
 - Expectation of quadratic regression coefficient is taken as 0.12 kg of 2. rice/wheat per kg of nitrogen.

3. RESULTS AND DISCUSSION

The yardsticks of additional production of rice* per tonne of Nranged between 10 and 13 units except at West Bengal where it was 8.6 units. The yardsticks of additional production per tonne of P2O_K were much less than those per tonne of N. It was least in West Bengal (3.8 units) and highest in Assam (10.4 units) and in other states between 6 and 9 units.

^{*} Cleaned rice.

The response of irrigated wheat to N and P₀O₅ and consequently the vardsticks of additional production were higher in general than those of rice, the variation being between 10-16 units for N and 8-10 units for PoOs.

On rice the year component of variation for the vardsticks of additional production from the use of nitrogen is quite large particularly in Andhra Pradesh, Assam and Kerala. The variation withinthe district in a given year, is an on average nearly twice the year to vear variation. The same is true of the yardsticks of additional production from the use of phosphorus. The year component of variation of the response to nitrogen was higher for irrigated wheat as compared to rice. The variation of the response to phosphorus from year to year was relatively smaller compared to variation within district.

The standard error of the estimated yardstick varied from 2.18 percent in Uttar Pradesh to 10.83 percent in Assam with nitrogen on rice and from 2.81 percent in Uttar Pradesh to 10.04 percent in Rajasthan on irrigated wheat. The variation with phosphorus ranged from 1.90 percent in Kerala to 19.15 percent in Madhya Pradesh on rice and from 2.86 percent in Bihar to 10.71 percent in Rajasthan on irrigated wheat.

The 95 percent confidence intervals for the yardsticks of additional production from the use of nitrogen were large in all states except at Bihar, Madras and Mysore for rice and at Uttar Pradesh for both rice and wheat. The confidence intervals for the yardsticks of additional production from the use of phosphorus are also large, the exceptions being Kerala for rice and Bihar for wheat.

The bias in the use of yardstick from the use of nitrogen was as high an overestimate as 7.8 units at Coimbatore. At Barabanki the bias is of the order of 0.5 units on rice and 4.6 units on irrigated wheat. With low rates of application as in Barabanki, the biases introduced by aggregation and standardisation of level are not likely to be large, but with rates of application as large as in Coimbatore. the situation is different.

SUMMARY

In agricultural planning fixation of targets is done through work schemes and supply schemes adopting Production Potential or Yardstic's approach The estimation of yardsticks of additional production from the use of fertilizers is done utilizing, large scale experimental data on cultivator's fields (where fertilizer treatments were superimposed on cultivators' normal practices) covering major rice and wheat-growing states, through standardising responses to diverse levels of nutrients using quadratic response curve.

The samping error, specification and aggregation errors affecting the yardsticks of additional production are discussed. The bias in the yardsticks of additional production due to specification and aggregation are worked out under three systems of weights. It is shown that the bias in the yardsticks can be removed under certain assumptions on the relationship between the size of the fields, rate of application of fertilizer, regression coefficients and also by suitable choice of the standard dose.

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